Analysis of Wave Propagation in Inhomogeneous Dielectric Slab Waveguides

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Abstract—**The propagation characteristics of the guided modes in an inhomogeneous dielectric slab waveguide are analyzed by the regularized WKBJ method. The corrected propagation constants of the guided modes in a near parabolic index medium are derived in analytic form. The effects of the refractive index profile and the homogeneous cladding on the guided modes can be expressed in terms of the mode indices and evaluated numerically.**

I. Introduction

An IMPORTANT aspect of the guided modes in a graded index multimode fiber involves the investigation of the propagation characteristics subjected to signal distortion caused by group delay difference among the guided modes. The Wentzel–Kramer–Brilloun (WKB) method is useful for analyzing this problem [1], [2]. The results are in good agreement with the exact solutions for the higher order modes but poor for the lower order modes in case of graded index fibers [3]. This method also gives reliable results for the optical waveguides formed by diffusion [4], [5]. There, however, is a negative assertion for the WKB method in which this method is neither applicable to a single-mode fiber nor to those modes which are close to cutoff [3], [6]. This mainly originates in the fact that the WKB solutions for the propagation constants of the lower order modes involve considerable errors [3]. To overcome this difficulty, a sophisticated method is proposed but it contains complex manifestations of the solutions [3]. Fortunately, we can find a simple closed-form method for refining the WKB approximation, henceforth referred to as a "regularized WKBJ method," in [7]. This method gives precise propagation constants asymptotically. The first-order term of the asymptotic expansion in the regularized WKBJ method is the WKBJ approximation with the corrected solutions of the Bohr–Sommerfeld quantum conditions.

The purpose of this paper is to show the usefulness of the regularized WKBJ method with application to the wave propagation in inhomogeneous dielectric waveguides. We analyze an inhomogeneous slab waveguide with a power law profile. The corrected propagation constants are represented in analytic form. The effects of the refractive index profile and the homogeneous cladding on the guided modes are explicitly expressed through the perturbed turning points or the propagation constants. As a result, this formulation may simplify the manifestation of the solutions and accompany with the clearer interpretation of the physical behaviors of the guided modes. These results are confirmed more in numerical examples. The time factor $\exp(-j\omega t)$ is suppressed throughout this paper.

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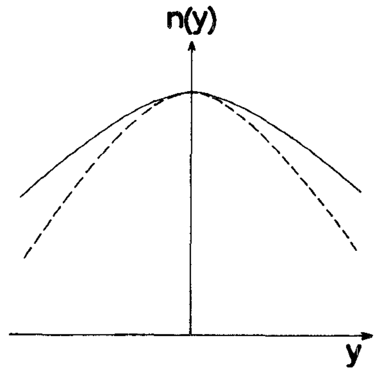

Fig. 1. The refractive index profile. ------ unperturbed, ——— perturbed.

II. Modes in an Uncladded Waveguide

We consider the wave propagation in an inhomogeneous dielectric slab waveguide with a near parabolic index profile

$$n(y) = n_0(1-\chi(y))^{1/2} \qquad \chi(y) = (gy)^2 - \alpha(gy)^4 + \beta(gy)^6 \tag{1}$$

where n_0 is the refractive index at the center of the waveguide, g is a positive constant, and α, β are constants (see Fig. 1). We develop the formulation for the TE-wave propagation along such a waveguide, since the TM wave can be handled in the same way [3]. We now write the mode function and the propagation constant in this waveguide as

$$\Phi_n(y)\exp(j\beta_n z) \qquad \beta_n = k(1-b_n)^{1/2}, \qquad n = 0,1,2,\cdots \tag{2}$$

where k is the wavenumber at $y=0$. The transverse mode function $\Phi_n(y)$ obeys the differential equation

$$\Phi_n''(y) + k^2(b_n - \chi(y))\Phi_n(y) = 0 \tag{3}$$

where the double prime denotes the second-order derivative with respect to y.

To obtain the solutions of (3), we use the regularized WKBJ method. First, let us describe the WKBJ solutions

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of (3). Under the WKBJ approximation, there exist the eigenvalues b_n such that

$$b_n = b_n^{(1)} + O(1/k^2), \qquad n=0,1,2,\cdots \tag{4}$$

where $b_n^{(1)}$ satisfies the Bohr–Sommerfeld quantum conditions

$$k\int_{-y(b_n^{(1)})}^{y(b_n^{(1)})} \left(b_n^{(1)} - \chi(y)\right)^{1/2} dy = (2n+1)\pi/2, \quad n=0,1,2,\cdots \tag{5}$$

with

$$b_n = \chi(y(b_n)), \qquad n=0,1,2,\cdots. \tag{6}$$

The points $-y(b_n)$ and $y(b_n)$ are called the turning points. The function $b_n - x(y)$ is linear in y over a suitable region near the turning point. Then the bounded WKBJ solution of (3) can be represented in terms of the Bessel function and the modified Bessel function of order $\pm 1/3$ [8].

Next, let us derive the formula for determining the second-order terms of the eigenvalues. The result is as follows (see Appendix) [7]:

$$b_n = b_n^{(1)} + b_n^{(2)}/k^2 + O(1/k^4), \qquad n=0,1,2,\cdots$$

$$b_n^{(2)} = (1/12)\left[d^2\left(\int_{-y(b)}^{y(b)} \left((d\chi(y)/dy)^2 \middle/ (b-\chi(y))^{1/2}\right) dy\right) \middle/ db^2 \right]_{b=b_n^{(1)}}$$

$$\bigg/ \int_{-y(b_n^{(1)})}^{y(b_n^{(1)})} \left(b_n^{(1)} - \chi(y)\right)^{-1/2} dy, \qquad n=0,1,2,\cdots. \tag{7}$$

From (7) together with (5) we can evaluate the corrected propagation constants with the error of order $1/k^3$.

The WKBJ method gives the exact solutions of (3) at the turning points and the asymptotic solutions with the error of order $1/k$ elsewhere [7]. Thus we can well represent the guided modes in this waveguide in terms of the first-order regularized WKBJ solutions, that is, the WKBJ solutions with the corrected propagation constants.

Last, let us calculate the propagation constants and the turning points of the guided modes in the waveguide with a near parabolic index profile (1). From (5), we can easily obtain the turning points in the form

$$\left(y(b_n^{(1)})\right)^2 = (y_{n0})^2\left(1 + (5/8)\alpha g^2 (y_{n0})^2 + \left((63/64)\alpha^2 - (11/16)\beta\right) g^4 (y_{n0})^4 + \cdots\right) + \begin{cases} 0, & \text{for the TE wave} \\ -(2-3\alpha) g^2 (y_{n0})^2/k^2, & \text{for the TM wave} \end{cases} \tag{8}$$

where y_{n0} denote the turning points of the square law medium:

$$y_{n0} = \left((2n+1)/kg\right)^{1/2}, \qquad n=0,1,2,\cdots. \tag{9}$$

Substituting (8) into (6), we have the propagation constants

$$\begin{aligned}(\beta_n^{(1)})^2 &= k^2(1-b_n^{(1)}) \\ &= k^2 - (2n+1)kg + (3/8)\alpha g^2 (2n+1)^2 \\ &\quad + \left((17/64)\alpha^2 - (5/16)\beta\right) g^3 (2n+1)^3/k + \cdots \\ &\quad + \begin{cases} 0, & \text{for the TE wave} \\ -g^2 - (2-3\alpha) g^3 (2n+1)/k, & \text{for the TM wave.} \end{cases}\end{aligned} \tag{10}$$

From (7) and (10), we can obtain the corrected propagation constants

$$(\beta_n)^2 = (\beta_n^{(1)})^2 + (3/8)\alpha g^2 + \left((67/64)\alpha^2 - (25/16)\beta\right) g^3 (2n+1)/k + O(1/k^2), \qquad n=0,1,2,\cdots. \tag{11}$$

The corresponding turning points are

$$(y(b_n))^2 = \left(y(b_n^{(1)})\right)^2 - (3/8)\alpha/k^2 - \left((19/64)\alpha^2 - (25/16)\beta\right) g^2 (y_{n0})^2/k^2 + O(1/k^4), \qquad n=0,1,2,\cdots. \tag{12}$$

The propagation constants (11) have been derived by the sophisticated method in case of $\beta=0$ [3]. For the refractive index with $\alpha=2/3$ and $\beta=17/45$, the propagation constants are

$$(\beta_n)^2 = (k-(n+1/2)g)^2 + \begin{cases} (1/4) g^2, & \text{for the TE wave} \\ -(3/4) g^2, & \text{for the TM wave.} \end{cases} \tag{13}$$

From this we can conclude that the refractive index profile with $\alpha=2/3$ and $\beta=17/45$ is not optimal in the sense of the equalization of the group delay difference among the guided modes, because of $\partial^2\beta_n/\partial n \partial k \neq 0$. It, however, is noted that $\partial^2\beta_n^{(1)}/\partial n \partial k = 0$.

III. Modes in a Cladded Waveguide

In this section we describe the guided modes in a graded index fiber with the homogeneous cladding. The refractive index $n(y)$ varies with the following form:

$$n(y) = \begin{cases} n(y), & y \leqslant y_a \\ n_0 n_a, & y > y_a \end{cases} \tag{14}$$

where n_a is a positive constant and y_a is the core boundary. For simplicity, we consider the case of the TE wave. It is easily supposed that the propagation constants of the guided modes in this waveguide may change their forms from β_n for the uncladded waveguide to β_v[9]:

$$\beta_v = k(1-b_v)^{1/2}, \qquad v = n + \Delta v, \qquad n=0,1,2,\cdots \tag{15}$$

with

$$b_v = (y(b_v)), \qquad n=0,1,2,\cdots. \tag{16}$$

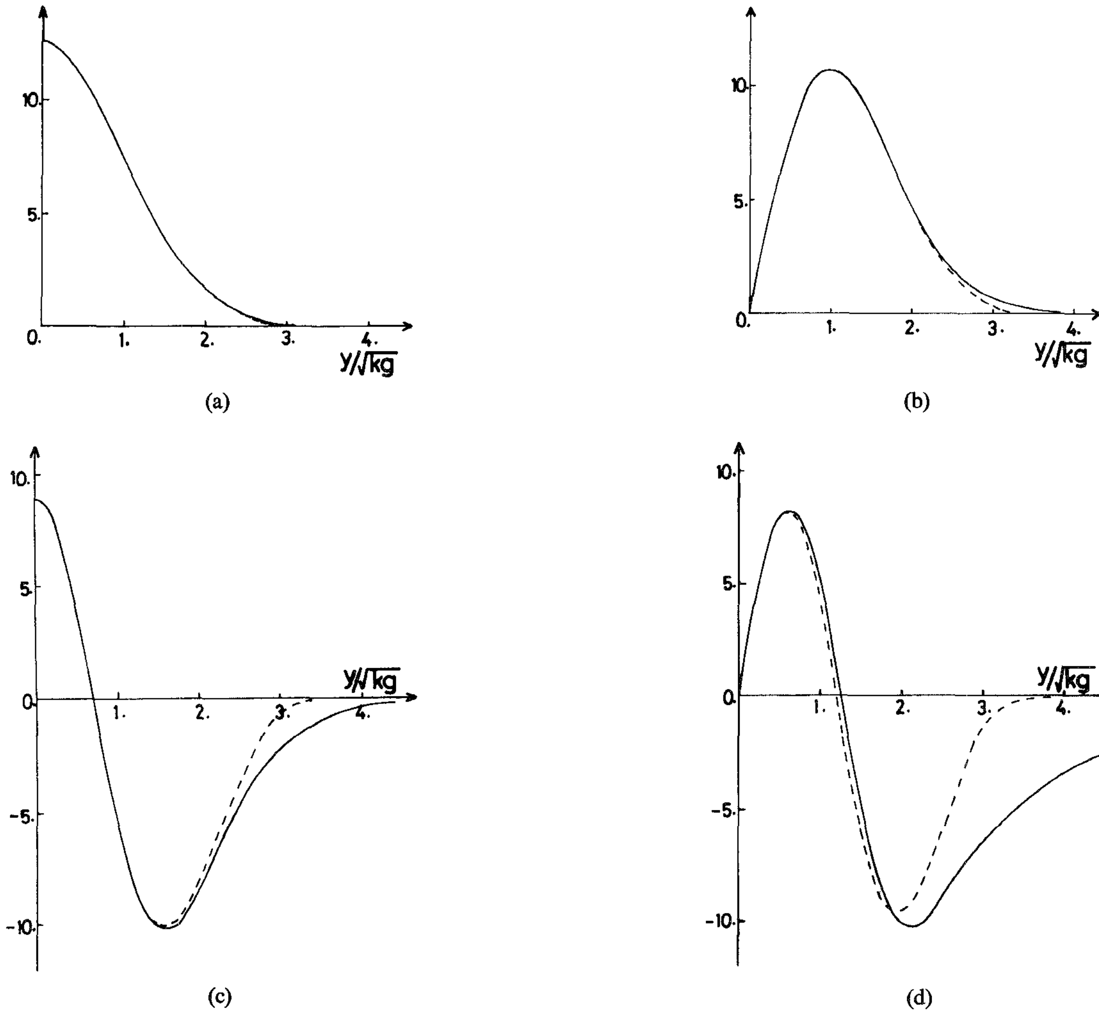

Fig. 2. Effect of the refractive index profile on the field distributions of the guided modes: TE wave with the parameters $k=1.53\times10^4/\text{mm}$, $g=3.23/\text{mm}$.------ $\alpha=\beta=0$, ——— $\alpha=100$, $\beta=0$. (a) $n=0$, (b) $n=1$, (c) $n=2$, (d) $n=3$.

TABLE I
VALUES OF D (PERCENT): COMPARISON WITH REGULARIZED WKBJ SOLUTIONS AND WKBJ SOLUTIONS WITH RESPECT TO THE PROPAGATION CONSTANTS; $k=1.53\times10^4/\text{mm}$, $g=3.23/\text{mm}$

n	$\alpha=\frac{2}{3}$ $\beta=\frac{17}{45}$		$\alpha=100$ $\beta=4000$	
	TE	TM	TE	TM
0	50.0	50.0	50.4	51.0
1	10.0	16.0	10.4	10.4
2	3.8	4.5	4.1	4.1
3	2.0	2.2	2.2	2.2

Then the electric field for the nth-guided mode can be represented in the form using two independent solutions Φ_v^1, Φ_v^2 of (3) with v

$$E_x(y,z)=\begin{cases}\left(A\Phi_v^1(y)+B\Phi_v^2(y)\right)\exp(j\beta_v z), & y\leqslant y_a\\ \left(A\Phi_v^1(y_a)+B\Phi_v^2(y_a)\right) & \\ \exp(-Q_v(y-y_a)+j\beta_v z), & y>y_a\end{cases} \tag{17}$$

where A and B are constants, and

$$Q_v=\left((\beta_v)^2-(kn_a)^2\right)^{1/2}, \qquad \text{Re}(Q_v)>0, \qquad \text{Im}(Q_v)<0.$$

The symbols Re( ) and Im( ) denote the real part of and the imaginary part of the quantity in parentheses. To determine the unknown numbers Δv, we use the boundary condition at $y=y_a$. The tangential component of the electric field is already matched. Therefore the tangential component of the magnetic field $H_z(y,z)$ must be matched at $y=y_a$. From this, and the symmetry property of the problem, we can obtain the characteristic equations

$$\Delta v=(2/\pi)\tan^{-1}\left[\left(\Phi_v^{1\prime}(y_a)+Q_v\Phi_v^1(y_a)\right)\Big/\left(\Phi_v^{2\prime}(y_a)+Q_v\Phi_v^2(y_a)\right)\right] \tag{18}$$

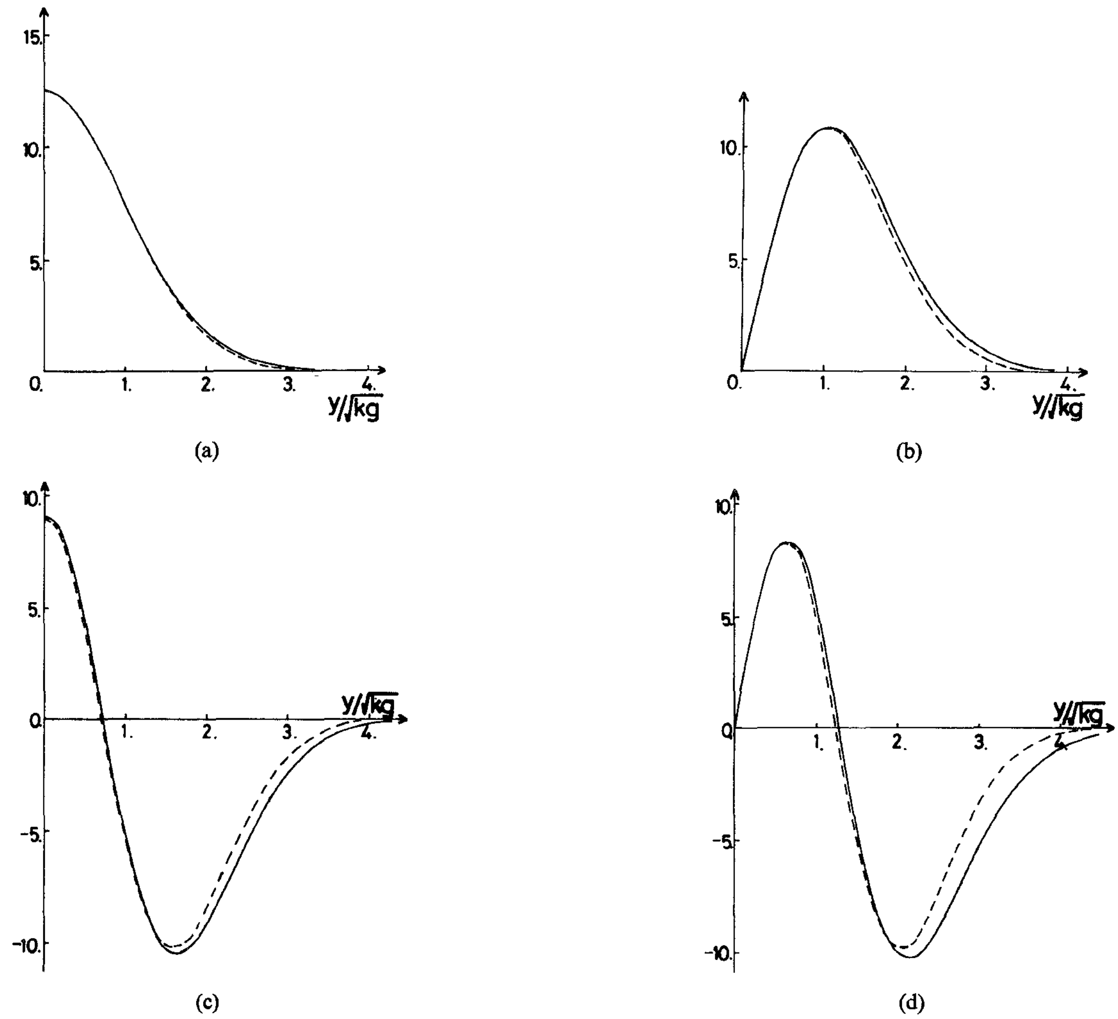

Fig. 3. Field distributions of the guided modes in inhomogeneous waveguide with homogeneous cladding: TE wave with the parameters $k=1.53\times10^4$/mm, $g=3.23$/mm, $y_a=1.26\times10^{-2}$/mm ($n(y_a)=1.5287$), $\alpha=\beta=0$. —— $n_0n_a=1.5288$, ------ $n_0n_a=1.5260$. (a) $n=0$, (b) $n=1$, (c) $n=2$, (d) $n=3$.

TABLE II

VALUES OF Δv FOR THE VARIOUS REFRACTIVE INDICES n_0n_a OF THE HOMOGENEOUS CLADDING: TE WAVE WITH THE PARAMETERS $k=1.53\times10^4$/mm, $g=3.23$/mm, $y_a=1.26\times10^{-2}$/mm ($n(y_a)=1.5287$), $\alpha=\beta=0$.

n \ n_0n_a	1.5260	1.5288	1.5290
0	-0.321×10^{-3}	0.655×10^{-4}	0.132×10^{-3}
1	-0.499×10^{-2}	0.127×10^{-2}	0.306×10^{-2}
2	-0.326×10^{-1}	0.132×10^{-1}	0.314×10^{-1}
3	-0.121	0.128	$0.139+j0.319$

where the prime denotes the derivative with respect to y. The procedure for determining the guided modes is summarized as follows.

1) The perturbed numbers Δv are numerically given as the solutions of the transcendental equations (18).

2) Substituting these into (15) and (16) in which indices n are replaced by v, we can calculate the propagation constants β_v and the turning points $y(b_v)$.

3) From (17) we can calculate the field distributions of the guided modes with the aid of the regularized WKBJ solutions.

Thus the regularized WKBJ method describes the guided modes in the cladded slab waveguide as simple as those in the uncladded waveguide.

IV. NUMERICAL EXAMPLES AND DISCUSSION

In order to confirm the validity of the present method, we provide several numerical examples.

First, let us consider the propagation constants of the guided modes in a near parabolic index medium. To evaluate the effect of the correction terms on the WKBJ solutions for the propagation constants, we introduce the quantity

$$D=\left((3/8)\alpha g^2+\left((67/64)\alpha^2-(25/16)\beta\right)g^3(2n+1)/k\right)$$
$$/\left((\beta_n)^2-k^2+(2n+1)kg\right)\times 100. \quad (19)$$

Table I shows that the correction terms are more significant to the lower order modes and the WKBJ solutions are in good agreement with the regularized WKBJ solutions for the higher order modes. In case of square law medium, the first-order term of the asymptotic expansion in the regularized WKBJ solution is equivalent to the WKBJ solution. In this case the field distributions of the guided modes calculated by the WKBJ method are compared with the exact ones. The results show the relative errors which in all cases are less than 1 percent. Therefore, the regularized WKBJ method well represents the guided modes in a near parabolic index medium, because this method gives the precise propagation constants or the turning points of the guided modes and the exact solutions at the turning points.

Fig. 2 shows the effect of the fourth-order term of the refractive index gradient on the field distributions. This effect is more significant to the higher order modes, Equations (8), (9), and (12) also imply this feature. To compare this effect with the cladding effect, we show Fig. 3. The solid lines in Figs. 2 and 3 are corresponding to each other physically. In fact, the turning points are shifted outwards in both cases. As a result, the energy carried by the guided modes tends to leak out. The effect of the outer layer on the guided modes is also significant to the higher order modes; this is also shown in Table II.

V. Conclusion

We solve the problems of the wave propagation in a near parabolic index medium without or with the homogeneous cladding in terms of the regularized WKBJ method. The corrected propagation constants are derived in analytic form. The wave propagation in a graded index slab waveguide may be well interpreted by considering the fact that the position of the turning point shifts according to the refractive index profile and the homogeneous cladding. The regularized WKBJ method may be applicable to the wave propagation in a radially inhomogeneous optical fiber.

VI. Appendix

We derive the formula (7) referring to [7]. Let $W_n(y)$ be the function which satisfies the equation

$$W_n''(y)+k^2(\mu_n-y^2)W_n(y)=0 \qquad \int_{-\infty}^{\infty} W_n(y)^2 dy=1 \quad (A.1)$$

where $\mu_n=(2n+1)/k$. Next, we construct the function such as

$$F_n(y)=\phi(y)W_n(\eta(y))\zeta(y) \quad (A.2)$$

where $\phi(y)$ is a function which takes zero outside $(-y(b)-2\delta, y(b)+2\delta)$ and unity inside $[-y(b)-\delta, y(b)+\delta]$, and $\eta(y)$ satisfies the equation

$$\eta'(y)=\left(\left(b_n^{(1)}-\chi(y)\right)/\left(\mu_n-\eta^2\right)\right)^{1/2} \quad (A.3)$$

and

$$\zeta(y)=(\eta'(y))^{-1/2}. \quad (A.4)$$

From (A.3) and (5), we have

$$\eta\left(y\left(b_n^{(1)}\right)\right)=(\mu_n)^{1/2}, \qquad n=0,1,2,\cdots. \quad (A.5)$$

Moreover, if $\chi(y)$ is m-times continuously differentiable at $y=\pm y(b_n^{(1)})$ and $\chi'(\pm y(b_n^{(1)}))\neq 0$, then the function $\eta(y)$ is also m-times continuously differentiable at those points.

Now the function $F_n(y)$ satisfies the equation

$$F_n''(y)+k^2\left(b_n^{(1)}-\chi(y)\right)F_n(y)-(\zeta''/\zeta)F_n(y)$$
$$=-\phi''\zeta W_n+2\left[\phi'(\zeta W_n)\right]', \qquad n=0,1,2,\cdots. \quad (A.6)$$

In the subdomain of $b_n^{(1)}<\chi(y)$, ϕ' and ϕ'' take nonzero values and $W_n(y)$ is the order of $\exp(-k\delta)$. Therefore, the right hand side of (A.6) is the order of $\exp(-k\delta)$. Multiplying (A.6) by $\Phi_n(y)$ in (3) and integrating by parts, we have

$$k^2\left(b_n-b_n^{(1)}\right)\int_{y^1-2\delta}^{y^2+2\delta}\Phi_n(y)F_n(y)\,dy$$
$$=-\int_{y^1-2\delta}^{y^2+2\delta}(\zeta''/\zeta)\Phi_n(y)F_n(y)\,dy+O(\exp(-k\delta)) \quad (A.7)$$

where we use the abbreviations as $y^1=-y(b_n^{(1)})$ and $y^2=y(b_n^{(1)})$. Next, let us evaluate the integrals in (A.7). To do this we utilize the asymptotic formula such that

$$\Phi_n(y)=P^{-1/2}\cos\left(k\int_{y^1}^{y}P\,dy-\pi/4\right) \quad (A.8)$$

where $P=(b_n^{(1)}-\chi(y))^{1/2}$. If $\Phi_n(y)$ is normalized as well as $W_n(y)$, then we can obtain the evaluation

$$\int_{y^1+\delta}^{y^2-\delta}\Phi_n(y)^2 dy=(2/T)\int_{y^1+\delta}^{y^2-\delta}(1/P)\cdot\sin^2\left(k\int_{y^1}^{y}P\,dy+\pi/4\right)dy$$
$$=1+O(\delta) \quad (A.9)$$

where

$$T=\int_{y^1}^{y^2}(1/P)\,dy.$$

The similar evaluation can be established for $F_n(y)$. Therefore, we have

$$k^2\left(b_n-b_n^{(1)}\right)=-\int_{y^1+\delta}^{y^2-\delta}(\zeta''/\zeta P)\sin^2\left(k\int_{y^1}^{y}P\,dy+\pi/4\right)dy$$
$$\Big/\int_{y^1+\delta}^{y^2-\delta}(1/P)\sin^2\left(k\int_{y^1}^{y}P\,dy+\pi/4\right)dy$$
$$+O(\exp(-k\delta))+O(\delta)$$
$$=-(1/T)\int_{y^1+\delta}^{y^2-\delta}(\zeta''/\zeta P)\,dy+O(\exp(-k\delta))+O(\delta). \quad (A.10)$$

From (A.3) and (A.4) we have the identity

$$\begin{aligned}(\zeta''/\zeta P) = &-(1/12)\Big[d^2\big((\chi')^2/(b-\chi(y))^{1/2}\big)/db^2\Big]_{b=b_n^{(1)}}\\ &+(1/4)d(\chi'/P^3)/dy\\ &-(1/2)\eta'(\mu_n-\eta^2)^{-3/2}-(5/4)\eta^2\eta'(\mu_n-\eta^2)^{-5/2}.\end{aligned}$$

Using this identity, we can easily obtain the relation

$$\int_{y^1+\delta}^{y^2-\delta}(\zeta''/\zeta P)\,dy = -(1/12)\cdot\left[d^2\left(\int_{y^1(b)+\delta}^{y^2(b)-\delta}\big((\chi')^2/(b-\chi(y))^{1/2}\big)dy\right)/db^2\right]_{b=b_n^{(1)}}+O(\delta). \tag{A.11}$$

From (A.10) and (A.11), and the limiting process $k \to \infty$ and $\delta \to 0$, we have the formula (7).

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The Electromagnetic Fields and the Phase Constants of Dielectric Image Lines

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Abstract—**A method is described for the exact calculation of the field distributions and the phase constants of single and coupled dielectric image lines of rectangular cross section. Field distributions and phase constants calculated by this method are presented as well as experimental results from lines fabricated of paraffin wax. The physical properties of the electromagnetic fields and the mode designation are discussed. The theory is compared to approximate calculation methods known from the literature.**

I. INTRODUCTION

DIELECTRIC IMAGE LINES are used as a basis of integrated millimeter-wave circuits; it is hoped that they will solve the problems which are known in connection with the application of microstrip lines in the millimeter-wave range. Therefore, more attention has been paid to this kind of microwave guide in the last five years by several authors; furthermore, the dielectric waveguide has been proposed for application in the optical range. Papers by Goell [1] and Marcatili [2], which are based on investigations by Schlosser and Unger [3], shall be mentioned here. Goell and Marcatili have examined rectangular dielectric waveguides embedded in a second dielectric material; Goell calculated the waveguides by expanding the fields into cylindrical eigensolutions, whereas Marcatili described an approximate solution which was found to neglect the electromagnetic fields of certain field regions. Toulios and Knox [4] in 1970 applied the solutions of Marcatili to the problem of the dielectric image line and showed the possible applications of the line for millimeter wave techniques. Goell [1] only gave the solution of the field problem of one single line; Marcatili described an approximate solution for two coupled lines, which in a similar way has been used by Toulios and Knox. A paper by Levige, Itoh, and Mittra [5] was also based on Marcatili's fundamental approximation method.

In this paper an exact solution is presented for the calculation of the phase constant and the field distributions of one single or two coupled dielectric image lines of rectangular cross section. The method presented can be

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